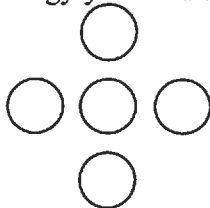
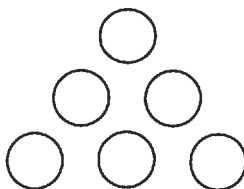


GRADE 4 OPEN-ENDED QUESTIONS

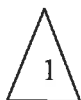
1. Place the digits 1, 2, 3, 4, and 5 in these circles so that the sums across and vertically are the same. Describe the strategy you used to find your solution(s).



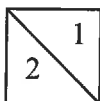
2. Levinson's Hardware has a number of bicycles and tricycles for sale. Johnnie counted a total of 60 wheels. How many bikes and how many trikes were for sale? Show how you got your answer in more than one way.
3. Melanie has a total of 48 cents. What coins does Melanie have? Is more than one correct answer possible?
4. Using each of 1, 2, 3, 4, 5, and 6 once and only once, fill in the circles so that the sums of the numbers on each side of the three sides of the triangle are equal. How was the strategy you used for problem #1 above similar to the strategy you used to solve this problem?



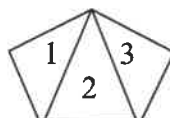
5. A rectangle has an area of 120 cm^2 . Its length and width are whole numbers.
- What are the possibilities for the two numbers?
 - Which possibility gives the smallest perimeter?
6. The product of two whole numbers is 96 and their sum is less than 30. What are the possibilities for the two numbers?
7. a. Draw the next three figures in this pattern:



3 sides



4 sides



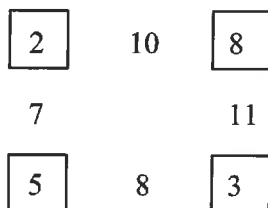
5 sides

- b. How many triangles are in a figure with 10 sides?

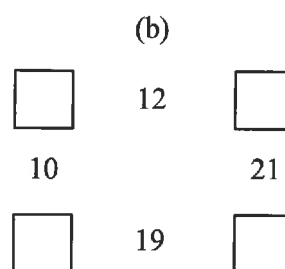
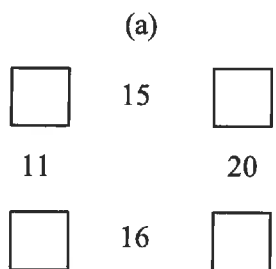
OPEN-ENDED QUESTIONS FOR MATHEMATICS

8. Study the sample diagram. Note that

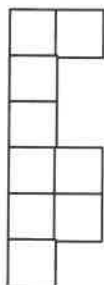
$$2 + 8 = 10 \quad 5 + 3 = 8 \quad 2 + 5 = 7 \quad 8 + 3 = 11.$$



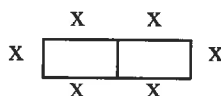
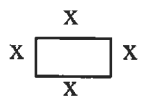
Complete each of these diagrams so that the same pattern holds.



9. Nine square tiles are laid out on a table so that they make a solid pattern. Each tile must touch at least one other tile along an entire edge. One example is shown below.



- What are the possible perimeters of all the figures that can be formed?
 - Which figure has the least perimeter?
10. In the school cafeteria, 4 people can sit together at 1 table. If 2 tables are placed together, 6 people can sit together.



- How many tables must be placed together in a row to seat: 10 people? 20 people?
- If the tables are placed together in a row, how many people can be seated using: 10 tables? 15 tables?

OPEN-ENDED QUESTIONS FOR MATHEMATICS

11. a. Fill in the blanks to continue this dot sequence in the most likely way.

\cdot $\cdot\cdot$ $\cdot\cdot\cdot$
 \cdot $\cdot\cdot$ $\cdot\cdot\cdot$ $\cdot\cdot\cdot\cdot$ _____
 \cdot $\cdot\cdot$ $\cdot\cdot\cdot$ $\cdot\cdot\cdot\cdot$ _____

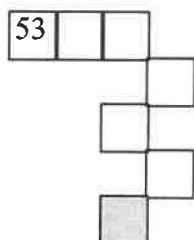
b. What is the number sequence for this pattern?

12. Here is the start of a 100 chart.

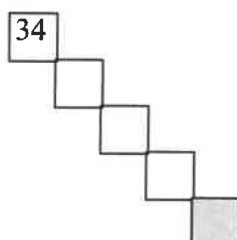
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

Shown below are parts of the chart. Without extending the chart, determine which numbers should go in the shaded squares.

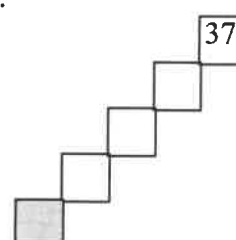
a.



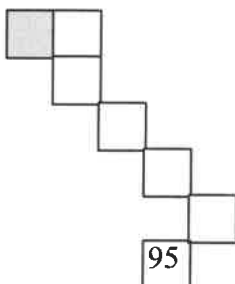
b.



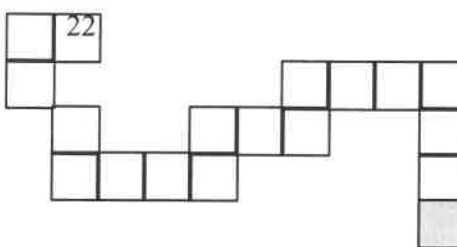
c.



d.



e.



13. The biggest animal in the world is the blue whale. Some blue whales have grown as long as 109 feet and have weighed 150 tons. A baby whale gains about 200 pounds a day.

a. How many pounds are in a ton?

b. How long will it take for a baby to weigh its first ton, if it was 400 pounds at birth?

OPEN-ENDED QUESTIONS FOR MATHEMATICS

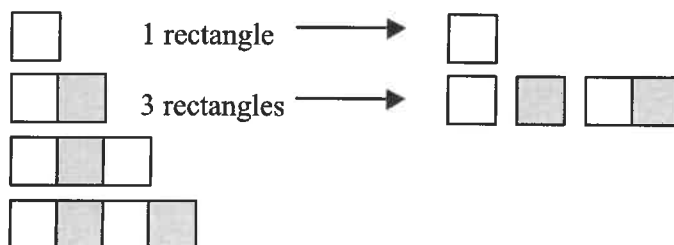
14. Ten white and ten red discs are lined up as shown.



Switching just two adjacent discs at a time, what is the least number of moves you can make to achieve the white, red, white, red, ... arrangement shown here? Explain the strategy you used to solve the problem.

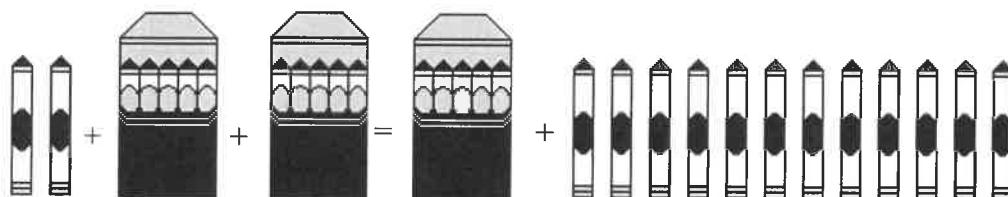


15. How many rectangles are there in each of these figures?



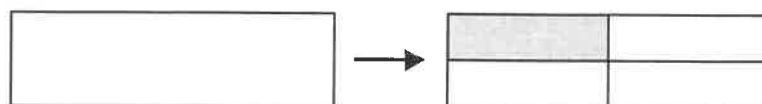
16. A basket starfish has more than 80,000 arms. If each arm needed a glove, how many pairs of gloves would you need?

17. How many crayons are there in each box? Each box has the same number of crayons and:

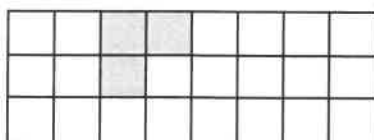


18. In a box of red, yellow, and blue color chips, all but 4 are red, all but 4 are yellow, and all but 4 are blue. How many color chips are in the box altogether?

19. One fourth of the rectangle on the left below is shaded by dividing it into fourths.

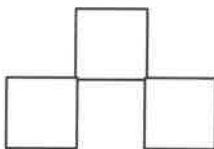


The shaded part below is one-fourth of some rectangle. Draw the rectangle.



OPEN-ENDED QUESTIONS FOR MATHEMATICS

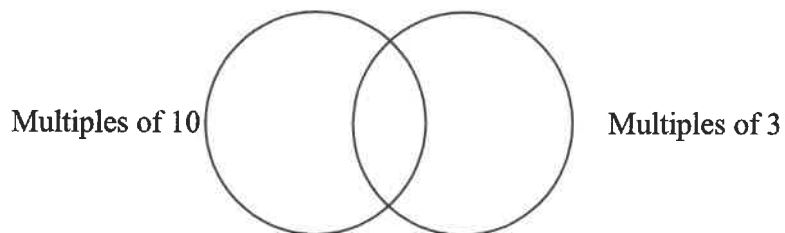
20. Three teachers have groups practicing different skits for open house. Mr. Jones has four groups containing 2, 3, 4, and 5 students. Mrs. Smith has groups of 4, 5, 6, and 7. Mrs. Philips has groups of 6, 7, 8, and 9. If each teacher wants to have the same number of students to supervise, then which group should be moved to another classroom? Explain your answer.
21. The castle gardener has a square for each type of rose. There are three roses in her garden. The queen has requested a new rose to be planted. If the gardener moves just three garden planks, one time only, she can get four squares. How can this be accomplished?



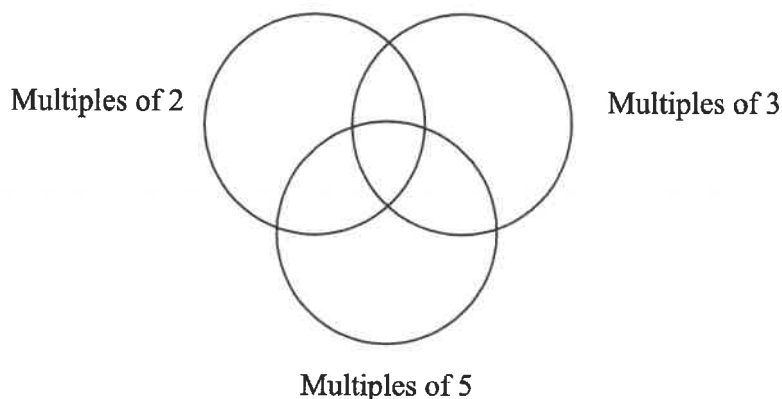
22. Jim has six American coins that total \$1.15. With his coins, Jim cannot give Ann exact change for a dollar. He cannot give Tim exact change for a fifty-cent piece. He cannot give Sean change for a quarter, or Jill change for a dime, or Cindy change for a nickel. What are Jim's six coins?
23. What is the greatest 3-digit number whose digits total 13? Justify your answer.
24. Sally and Jim each have a bag of hard candy. Sally said, "Jim, if you give me 5 pieces of candy from your bag, I'll have as many pieces as you." Jim laughed and answered, "No, you give me 5 of yours and I'll have twice as many as you." How many pieces did they each have to begin with?
25. A farmer fenced a square plot of ground. When he finished, he noted that there were five fence posts on each of the sides. How many posts are used to fence the plot?
26. Twice the product of 6×5 is three times as great as this number. What is the number?
27. Can you arrange four 5's so that they equal six? (Hint: You must use fractions.)
28. Why are 1997 pennies worth almost twenty dollars?
29. a. These numbers belong together in a group: 25, 40, 110, 55
These numbers do not belong in the group: 33, 71, 4, 106
Which of these numbers belong in the group? 75, 205, 87, 43
What is the rule?
- b. These numbers belong together in a group: 16, 9, 49, 64
These numbers do not belong in the group: 40, 12, 77, 28
Which of these numbers belong in the group? 102, 36, 25, 50
What is the rule?

30. Write the numbers in each circle that are described by the attribute written outside the circle. Numbers in overlapping regions must meet the requirements of **each** circle. Justify why you placed the numbers in the overlapping regions you did.

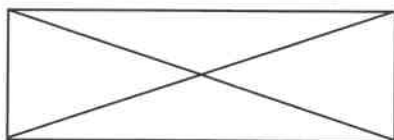
a.



b.



31. Jacob, Jon, and Amanda were building a rectangular snow wall one block thick. They made 24 blocks of snow and couldn't decide what the dimensions should be. Give them a list of the different size walls, using whole blocks.
32. a. How many triangles can you find in the figures below?
b. How many different edges are used in these triangles?
c. If the area of each of the 4 smallest triangles is the same and this area is 1 square unit, what is the area of each triangle in the figure?



33. John was looking through his math book and discovered that after page 44, the next page was 51. How many page sheets were torn out of his book? Provide an explanation that will justify your answer.

OPEN-ENDED QUESTIONS FOR MATHEMATICS

34. Bob's family of three was driving to Nashville. They were going to stay overnight, sightsee during the next day, and return home in the evening. They had to pay for dinner, breakfast, and lunch. They were to sleep at Grandma's house. Breakfast at McDonald's was \$2.32 each. Lunch at Kentucky Fried Chicken was \$3.29 each. Dinner at Wendy's was \$4.89 each. Was \$40 enough money to pay for their food?
35. Jill's mother limited her Nintendo playing to 10 hours per week. She played on only four days, a different amount of time each day. On Saturday, she played twice as much as on Wednesday. She didn't play on Monday, Tuesday, or Thursday. On Friday, she played the least of the days she played. If the times were all different and there were not any partial hours, how many hours did she play on each day?
36. To encourage John to work harder in math his mother said she would pay him 10 cents for each right answer and subtract 5 cents for each wrong answer. If he earned 20 cents after doing 32 problems, how many problems did John get right? How many did he get wrong? How many would he have to get right to earn more than a dollar?
37. Joan went fishing. On the first cast she hooked a fish 80 feet from the boat. Each time she reeled in 10 feet of line, the fish would take out 5 feet. How many times did she have to reel in to get the fish to the boat?
38. "If you have a square and you cut off one corner, how many corners do you have left?" asked Mrs. Wheeler. "Easy," answered Tony. "Three." "Wrong, Tony!" cried Donnie. Where did Tony go wrong? Explain (including a sketch).
39. An Egyptian pyramid has a square base and four triangular faces. Use clay or Play-Doh to make a pyramid model. If you make different "plane cuts" through all 4 of the triangles -- without cutting through the square -- which different shapes will the cuts make? A plane cut is kind of like a cheese cutter -- it will be straight, not curved.
40. Use the digits 0, 1, 2, 3, 4, 5, 6, 7 to find the smallest answer possible in this problem. No negative numbers are allowed.
- $$\begin{array}{r} \square \square \square \square \\ - \square \square \square \square \\ \hline \end{array}$$
41. Bill received \$12 to feed a neighbor's cat for 3 days. At this pay rate, how many days will he have to feed the cat to earn \$40? The neighbor's family is going on vacation for 3 weeks next summer. Bill wants to earn enough money to buy a CD player that costs \$89. Will he have enough money? Explain.

OPEN-ENDED QUESTIONS FOR MATHEMATICS

42. A farmer milks 16 cows every hour. The size of his herd varies as new calves are born and he sales off some of his stock. He never has less than 48 cows nor more than 64. How many hours does it take him to milk his cows? Explain.
43. Jane's mom needs to buy enough gas to fill up her car's tank. She will need 15 gallons. There is a Shell station 3 blocks from her home that sales the gas for \$1.19/gallon. There is a Speedway station one mile away that sales the gas for \$1.07/gallon. How much money will it cost her to buy gas at each station? How much will she save by going to Speedway?
44. Bradley works as a traveling salesman and leases a car for his business. The leasing company has a base rate for the first 15,000 miles per year, but has an additional charge of $\$500 + \$0.25/\text{mile}$ for each mile over the 15,000. Bradley has already driven 4300 miles in the first three months. If he drives a similar amount for the rest of the year, how much will he have to pay extra?
45. A clock loses 2 minutes every 8 hours. Alicia's mom plans to set the alarm at 11 p.m. on Sunday night to get Alicia up each morning. Alicia has to get up each day no later than 7 a.m. What time should her mom set the alarm to be sure that Alicia isn't late to school any day, Monday through Friday?

GRADE 4 SOLUTIONS

1. *Extend a pattern* (E-1.2.2) (E-4.2.1)
- | | | | | |
|-------|----|-------|----|-------|
| 2 | | 3 | | 2 |
| 1 3 5 | or | 2 1 5 | or | 1 5 4 |
| 4 | | 4 | | 3 |

2. *Recognize/extend/find rules for number patterns* (E-4.2.1, E-4.2.2, E-4.3.2)

<u>Bikes</u>	<u>Trikes</u>	<u>Bike Wheels</u>	<u>Trike Wheels</u>	<u>Total</u>
30	0	60	0	60
27	2	54	6	60
24	4	48	12	60
etc.				

3. *Recognize/extend/find rules for number patterns; add, subtract, multiply amounts of money* (E-4.2.1, E-4.2.2)

<u>Pennies</u>	<u>Nickels</u>	<u>Dimes</u>	<u>Quarters</u>	<u>Total</u>
48	0	0	0	48
43	1	0	0	48
38	2	0	0	48
38	0	1	0	48
33	1	1	0	48
33	3	0	0	48
etc.				

4. *Recognize/extend/find rules for number patterns* (E-4.2.1)

6		1
1 2	or	6 5
5 3 4		2 4 3

5. *Perimeter and area* (E-2.2.5; M-2.2.5)

a. If we assume that length must be longer than width, then the possibilities are:

Length	120	60	40	30	24	20	15	12
Width	1	2	3	4	5	6	8	10
Area	120	120	120	120	120	120	120	120
Perimeter	242	124	86	68	58	52	46	44

- b. 12 cm by 10 cm appears to be the rectangle with the smallest perimeter.
(Actually, the rectangle with the smallest perimeter is a square each of whose sides is $\sqrt{120}$, or 10.954451...)

6. *Factors* (E-1.2.7; M-1.2.4) 4 and 24, 6 and 16, 8 and 12.

7. *Identify/create patterns; extend/find rules for number patterns* (E-4.2.1, E-4.2.2, E-4.1.1) b. 8, or n-2

OPEN-ENDED QUESTIONS FOR MATHEMATICS

8. *Recognize/extend/find rules for number patterns* (E-1.1.5, E-1.2.2, E-4.1.1, E-4.2.1, E-4.3.1)

There are multiple solutions for each, for example:

$$\begin{array}{ccc} \text{(a)} & 7 & 15 & 8 \\ & 11 & & 20 \\ & 4 & 16 & 12 \end{array} \quad \text{or} \quad \begin{array}{ccc} 6 & 15 & 9 \\ 11 & & 20 \\ 5 & 16 & 11 \end{array} \quad \text{or} \quad \begin{array}{ccc} 4 & 15 & 11 \\ 11 & & 20 \\ 7 & 16 & 9 \end{array}$$

$$\begin{array}{ccc} \text{(b)} & 3 & 12 & 9 \\ & 10 & & 21 \\ & 7 & 19 & 12 \end{array} \quad \text{or} \quad \begin{array}{ccc} 2 & 12 & 10 \\ 10 & & 21 \\ 8 & 19 & 11 \end{array} \quad \text{or} \quad \begin{array}{ccc} 6 & 12 & 6 \\ 10 & & 21 \\ 4 & 19 & 15 \end{array}$$

9. *Perimeter* (E-2.2.5, E-1.1.5) a. 12, 14, 16, 18, 20 b. A square (3 x 3)
10. *Identify/create patterns in real-life situations; extend/find rules for number patterns* (E-4.1.1, E-4.2.1, E-4.3.1)
- 10 people = 4 tables, or $[(p-2)/2]$
20 people = 9 tables
 - 10 tables = 22 people, or $2t + 2$
15 tables = 32 people
11. *Recognize/extend/find rules for number patterns* (E-4.2.1, E-4.3.2)
- 2, 5, 8, 11, 14, ..., $3n-1$ (add 3 to find the next term)
12. *Recognize/extend/find rules for number patterns* (E-4.2.1)
- 95 b. 78 c. 73 d. 42 e. 70
13. *Use customary units of measure; multistep story problems using combination of operations; division by multiples of 10* (E-2.1.5, E-1.2.2)
- 2000 pounds in a ton
 - $2000 - 400 = 1600$; $1600 \div 200 = 8$ days
14. *Recognize/extend number patterns* (E-4.2.1, E-4.2.2, E-4.3.1)
- 10 (Note: The results are the triangular numbers, i.e., two of each color requires 1 switch; three of each color requires 3 switches; 4 of each color requires 6 switches; etc.)
15. *Recognize/extend number patterns* (E-4.2.1, E-4.2.2, E-4.3.1)
- 6 and 10
- (Note: The results are the triangular numbers, i.e., 1, 3, 6, 10, 15, ..., $\frac{n(n+1)}{2}$)
16. *Division by 1-digit divisor; division with 0's in quotient* (E-1.2.2)
- $80,000 \div 2 = 40,000$ pairs of gloves

OPEN-ENDED QUESTIONS FOR MATHEMATICS

17. *Explore the use of variables and open sentences to express relationships; multistep problems using a variety of operations* (E-4.2.3, E-4.1.2)

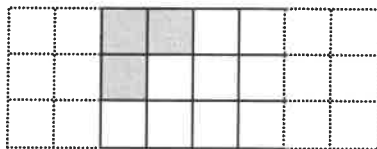
There are 10 crayons in each box.

18. *Draw conclusions/make inferences based on data* (E-3.1.3, E-3.3.1)

6 chips: 2 red, 2 yellow, 2 blue.

19. *Use models to solve real world problems involving fractions; relate fractions using models to represent equivalencies* (E-1.1.5; M-1.1.6)

One solution:

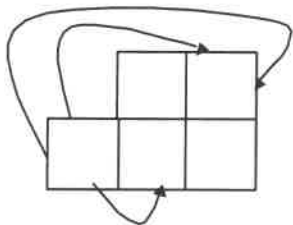


20. *Column addition; multistep story problems using combination of operations* (E-1.2.2)

Mr. Jones has 14 students; Mrs. Smith has 22 students; Mrs. Philips has 30 students.

Move the group of 8 from Mrs. Philips to Mr. Jones.

21. *Introduce transformations - translation (slide)* (E-2.2.3)



22. *Add, subtract, multiply amounts of money; make change; collect/organize/interpret data* (E-2.2.6, E-3.2.2)

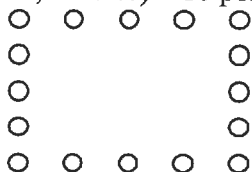
A half-dollar, a quarter, and four dimes

23. *Place value* (E-1.1.4, E-1.2.9, E-1.3.3) 940 ($9 + 4 + 0 = 13$)

24. *Multistep story problems using combination of operations* (E-4.2.3)

Sally had 25 pieces; Jim had 35.

25. *Perimeter.* (E-1.1.5, E-2.2.5, E-2.2.8) 16 posts.



26. *Multiplication facts; multiples of 10.* (E-1.2.2, E-4.2.3) 20

OPEN-ENDED QUESTIONS FOR MATHEMATICS

27. *Numerator/denominator; multistep problems; introduce improper fractions.*

(E-1.3.1, E-1.3.3; M-1.3.1)

$$\frac{55}{5} - 5 = 6$$

28. *Decimal place value with concrete models.* (E-1.1.4)

One thousand, nine hundred and ninety-seven pennies are worth \$19.97, which is almost \$20.

29. *Multiples/ square numbers; represent and describe mathematical relationships through looking for a pattern.* (E-1.1.3, E-4.2.1)

- a. 75; 205 -- multiples of 5
- b. 36; 25 -- square numbers

30. *Multiples* (E-1.1.3)

- a. Multiples of 10: 10, 20, 30, ...; multiples of 3: 3, 6, 9, 12, ...; multiples of both 3 and 10: 30, 60, 90, ...
- b. Multiples of 2: 2, 4, 6, 8, ...; Multiples of 3: 3, 6, 9, ...; multiples of 5: 5, 10, 15, ...; multiples of both 2 and 3: 6, 12, 18, ...; multiples of both 3 and 5: 15, 30, 45, ...; multiples of both 2 and 5: 10, 20, 30, ...; multiples of all 3 (2, 3, and 5): 30, 60, 90, ...

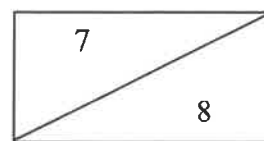
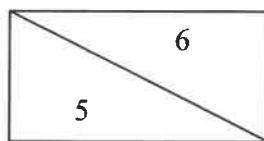
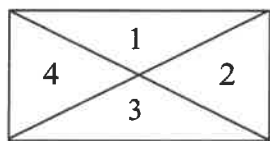
31. *Factor pairs.* (E-1.2.7)

Each wall is 1 block thick, then the remaining dimensions are:

1 block x 24 blocks; 2 blocks x 12 blocks; 3 blocks x 8 blocks; 4 blocks x 6 blocks

32. *Triangles; edges; area of triangle* (E-2.1.1)

a.



- b. 12 edges in the first figure above; 1 new edge in the second figure (both triangles share a commonly labeled edge); 1 new edge in the third figure -- a total of 14 edges.
- c. Area 1 = 1; area 2 = 1; area 3 = 1; area 4 = 1; area 5 = 2; area 6 = 2; area 7 = 2; area 8 = 2.

33. *Compare and order whole numbers; identify/create patterns in real-life situations.* (E-1.2.9)

3 pages: (45 & 46; 47 & 48; 49 & 50).

34. *Add, subtract, and multiply amounts of money.* (E-1.2.3) Yes.

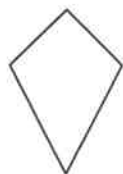
Breakfast was	3 x \$2.32 = \$6.96
Lunch was	3 x \$3.29 = \$9.87
Dinner was	3 x \$4.89 = \$14.67
Total	= \$31.50

OPEN-ENDED QUESTIONS FOR MATHEMATICS

35. *Multistep story problems involving combination of operations.* (E-1.2.2)
Jill played 1 hour on Friday, 2 hours on Wednesday, 3 hours on Sunday, and 4 hours on Saturday.
36. *Choosing the appropriate problem solving strategy; add/subtract/multiply amounts of money.* (E-1.2.3)
If he got 12 right and missed 20, then he earned 20 cents. He would need to have at least 18 right to earn more than a dollar.
37. *Identify/describe/create patterns in real-life situations.* (E-3.2.2) 15
38. *Vertices of angles.* (E-2.1.1) The sheet of paper originally had 4 vertices (corners). When a cut was made, it actually created an additional vertex. The sheet now has 5 vertices.



39. *Introduce trapezoid; parallel; (kite).* (M-2.1.2; E-2.1.1)
A square, trapezoid, and kite (quadrilateral in which there are exactly two pairs of congruent sides -- see figure below).
- 1) if the cut goes through all 4 triangles parallel to the base, a square is produced;
2) if the cut is made through one pair of opposite faces that are parallel to each other and to the base, then a trapezoid is produced;
3) if the cuts are made from one edge towards the opposite edge, which is a different distance from the base in such a way that the other two vertices are both equidistant from the base, then a kite is produced.



Note: Because of its difficulty, this problem would be better used as a performance event with students working together than as an open response question.

40. *Place value; add/subtract 3- and 4-digit numbers.* (E-1.1.4, E-1.2.2, E-1.3.3)

$$\begin{array}{r} 4012 \\ - 3765 \\ \hline 247 \end{array}$$

NOTE: The remaining problems are designed to be done in cooperative groups -- not for individual students.

41. *Multistep story problems; add/subtract/multiply amounts of money.* (E-1.2.3)
\$40 - 10 days. For 3 weeks (21 days), Bill will earn \$84 -- not enough for an \$89 CD player.

OPEN-ENDED QUESTIONS FOR MATHEMATICS

42. *Multistep story problems; choosing the appropriate problem solving strategy.* (E-4.2.1, E-1.2.2)
3 – 4 hours.
43. *Multistep story problems; add/subtract/multiply amounts of money.* (E-1.2.3)
Shell - \$17.85; Speedway - \$16.05; savings - \$1.80
44. *Multistep story problems; add/subtract/multiply amounts of money.* (E-1.2.3)
 $\$500 + \$550 = \$1050$
45. *Multistep story problems; choosing the appropriate problem solving strategy; add/subtract amounts of time.* (E-2.2.6)
13 eight-hour intervals between 11 p.m., Sunday and 7 a.m., Friday.
 $13 \times 2 = 26$ minutes. $7:00 - 0:26 = 6:34$ a.m. The alarm needs to be set for 6:34 a.m. to be sure that Alicia gets up by 7 a.m. on Friday.