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Math

Through these math lessons students investigate some of the subtler aspects of basketball while using group problem solving and powerful mathematical tools such as algebra, stem-and-leaf plots, symmetry, ratio, percentage and probability. Sports trivia, a fun game, challenging activities plus a mathematics contest add up to a nice change of pace without sacrificing mathematical content.

LESSON 1

Circumference of the Changing Basketball

Students will compare changing basketball sizes and shoot for a conclusion about circumference.

National Standards: NM.5-8.2, NM.5-8.3, NM.5-8.4, NM.5-8.8
Skills: Problem solving, using the formula for circumference
Estimated Lesson Time: 40 minutes

Teacher Preparation

- Duplicate the Circumference of the Changing Basketball worksheet on page 307 for each student.
- Collect the items listed under Materials.
- Select related problems for the homework assignment.

Materials

- 1 calculator for each group
- 1 copy of the Circumference of the Changing Basketball on page 307 for each student
- 1 ruler for each group
- 1 paper clip for each group
- 1 fat marker for each group
- 1 pencil for each student

Background Information

Basketball is the result of invention and experimentation. In 1891 Dr. James Naismith was assigned to find a winter sport for young men to play indoors at the YMCA in Springfield, Massachusetts. He came up with basketball, a game roughly patterned after a game he played as a boy. Naismith used a soccer ball because a large ball would be safer for indoor play; however, as the game spread any available ball was used—even a football.

The first basketball was manufactured in 1894. In 1930 the official size was reduced from a 32-inch circumference to 31 inches. Later, in 1934, the size was reduced again, to about 30 inches, its size today. The women's basketball, at 29 inches, is even smaller.

In this lesson we use the changes in circumference of a basketball as a starting point for a discussion about the relationship between circumference and radius. A problem is posed; then, in small cooperative groups, the students use their problem-solving skills to attack the problem. They tackle a similar problem using the 30-inch and 29-inch basketballs. The results are surprising because they are exactly the same.

Introduce the Lesson

Go over the history of the basketball. Tell students that they have an opportunity to make some discoveries about circumference and to gain experience problem solving as a group. Group problem solving is important in business, engineering and even some math contests.

Follow These Steps

1. Divide the students into groups of three or four and pass out the worksheet on page 307. Remind them to assign tasks: taskmaster (to watch the clock and keep everyone on track), note takers and presenters. All should contribute to solving the problems.
2. Tell students they have 20 minutes to solve the problems.
3. Allow students to work. Circulate.
4. After about 20 minutes, ask students to present their solutions. Do not worry if the third problem is not done. You can add it to the homework assignment.
5. Discuss and summarize the conclusions.
6. Assign related problems for homework.

Extend and Vary the Lesson

- Consider problem 3 on the worksheet about the circumference of the Earth. How would the answer change if the ring had been increased by a mile instead of a yard? By a foot instead of a mile?
- Explore how the change in size affected the manufacturers of the ball. How did the volume and surface area change? Figure cost differences in making the ball.
- Explore how the change in size affected the players. What are the advantages of a smaller ball? Why was the change made?

References

Gardner, M. 1961. *Entertaining Mathematical Puzzles.* New York: Dover.
Stewart, M. 1998. *Basketball: A History of Hoops.* New York: Franklin Watts.

Circumference of the Changing Basketball

Name____ Date______________________

Solve the following problems in order. Show your work and be prepared to present your answers. (Hint: Be sure to label units of measure.)

1. In 1934 the circumference of the basketball changed from 31 inches to 30 inches. How much of a difference did this make? To compare the size of the 30-inch and 31-inch basketballs, imagine that a steel band of 31 inches is placed around the middle of a 30-inch ball in such a way that the distance between the ball and the band is constant. What is the thickest item that can be slipped between the band and the ball—a paper clip, a plastic ruler, a pencil or a fat marker? Show how you arrived at your answer.

2. The women's basketball is about 29 inches in circumference. Suppose it is exactly 29 inches. Compare it to the 30-inch ball. Repeat problem 1 using a 29-inch ball and a 30-inch band. What do you notice?

3. Suppose a band exactly 1 yard longer than the circumference of the Earth is placed around the Earth in such a way that it remains a constant distance from the equator. Assume the Earth is smooth and that its equator is 7,926 miles. Could a human pass under the ring?

4. How are the solutions for the three problems alike? How are they different?

LESSON
2

Creating and Destroying Symmetry

Students will reflect on the types of symmetry that appear in different basketball symbols and point out changes that will rotate the symmetry from one type to another.

National Standards: NA-NM.5-8.12, NA-NM.5-8.3, NA-NM.5-8.4
Skills: Identifying line, point and rotational symmetry; changing the type of symmetry in a design by adding or erasing lines or regions in the design; discovering which elements in a design contribute to different types of symmetry
Estimated Lesson Time: 45 minutes

Teacher Preparation

- Duplicate the Creating and Destroying Symmetry worksheet on pages 312-313 for each student.
- Print copies of the symbols *N, S* and *A* using a Helvetica font and the largest font size available. If 72 is the largest font size available using your computer, print that size and then enlarge the letters using a photocopier. Cut the symbols apart so you can hold each one up separately. You can turn these props upside down to illustrate which letters have point symmetry and which do not.
- Collect the items listed under Materials.

Materials

- 1 copy of the Creating and Destroying Symmetry worksheet on pages 312-313 for each student
- 1 pencil for each student
- 1 bottle of correction fluid for each pair of students

Background Information

In his book *Symmetry*, mathematician Herman Weyl wrote, “Symmetry, as wide or as narrow as you may define its meaning, is one idea by which man through the ages has tried to comprehend and create order, beauty and perfection.” Our idea of symmetry ranges from a general idea of harmony and proportion to more specific ideas such as line, point, rotational, bilateral, crystallographic and mathematical symmetry. In this lesson we will look at the symmetry of the basketball court from a mathematical point of view. We will see that mathematicians precisely define the different symmetries and work to discover the relationships involved. The following is a brief intuitive introduction to the way mathematicians view three types of symmetry: line, rotational and point.

To test for *line symmetry,* draw the possible line of symmetry, fold the picture along the line, crease and hold the picture up to the light to see if the two sides match. If they match perfectly, the shape has line symmetry. The line of symmetry is the perpendicular bisector of the segment joining any point to the point it would match up with when the picture is folded along the line of symmetry.

To test for *rotational symmetry*, locate a point that appears to be the center of the shape. Using tracing paper, trace the shape and the center. Keep the two centers lined up. Rotate the tracing paper until the two pictures line up again. If they line up again in less than one 360-degree turn, the shape has rotational symmetry.

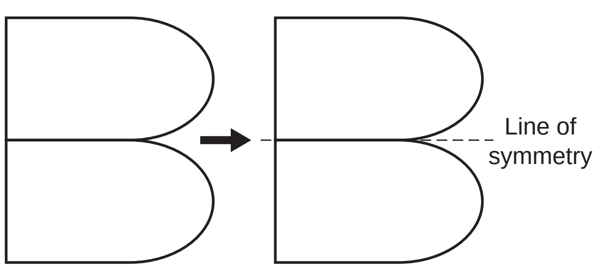
A set of points has *point symmetry* if there exists a center point, C, such that every point in the set has an image (i.e., a partner also in the set) and C is the midpoint of the segment joining the two points. If C is in the set, C is its own partner. Point symmetry is the same as a rotation of 180 degrees, so any picture with point symmetry also has rotational symmetry. A picture has point symmetry if it looks exactly the same upside down as it does when it is right-side up.

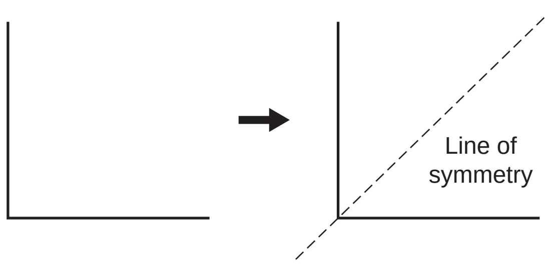
Introduce the Lesson

Remind students that symmetry is useful not only in art and design, but also in biology, chemistry, physics and mathematics. When they are in high school they will learn to lessen their work in graphing equations by first identifying whether the graph should have symmetry. In this lesson they will develop their understanding of symmetry by adding or erasing lines in a design to change it from one type of symmetry to another.

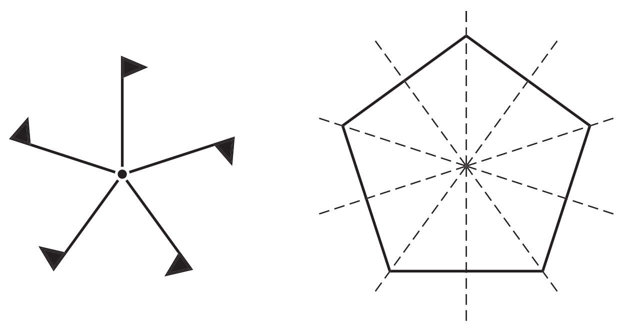
Follow These Steps

1. If you have not discussed symmetry with your class, review the definitions of the three types of symmetry. If you have recently talked about symmetry, begin the lesson and incorporate the definitions.
2. Write "NCAA BASKETBALL" on the board and ask the students to identify the kinds of symmetry in each letter. Be careful to draw each with the appropriate symmetry. (*The letters* C, E, A *and* T *all have line symmetry.* C *and* E *have a horizontal line of symmetry whereas* A *and* T *both have a vertical line of symmetry. The letters* N *and* S *have both point and rotational symmetry. We know they have point symmetry because they look the same upside down as they do right-side up.*) Remember that any design with point symmetry has rotational symmetry since point symmetry is a rotation of 180 degrees about the center point of the design. What about the letter *B*? It would have a horizontal line of symmetry if we made the bulge at the top the same size as the bulge at the bottom. The letter *L* (as written) has no symmetry; however, if we extended the horizontal bar so it was exactly as long as the vertical bar, it would have a diagonal line of symmetry! Ask how the letter *K* should be changed to produce symmetry. (*The diagonal bars should be the same size and cross at the midpoint of the vertical bar.*)

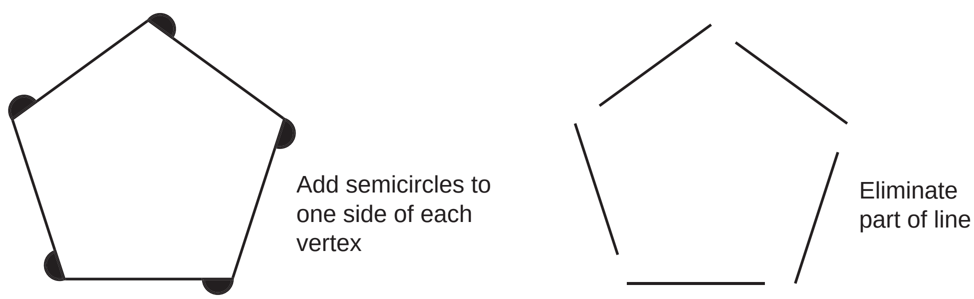




3. Ask if any letters of the alphabet have more than one line of symmetry. (*The letters* H, I, O *and* X *all have both a horizontal and a vertical line of symmetry. Written to look like a circle,* O *has infinitely many lines of symmetry and each line passes through the center of the circle.*)
4. Say that every letter that has point symmetry also has rotational symmetry, but that some designs have only rotational symmetry. Ask them to think of a design that contains only rotational symmetry. (*Possible correct answers include a pinwheel. Be careful; many pictures that have rotational symmetry, but not point symmetry, also have line symmetry. Draw a regular pentagon. Point out to the students that a regular pentagon has rotational symmetry plus five lines of symmetry.*)



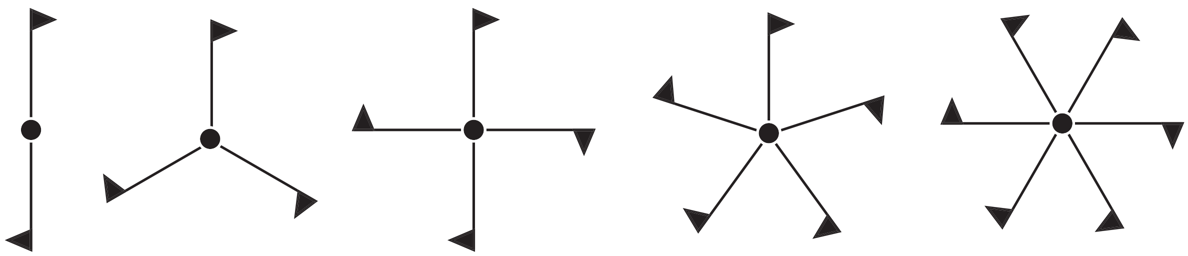
5. Ask the students if it is possible to add or erase something in the picture of the pentagon so as to eliminate the lines of symmetry.



6. Ask the students what happens to the pinwheel if we eliminate the little flags at the end of each pinwheel. (*We get line symmetry as well as rotational symmetry.*)
7. Distribute the worksheet on pages 312-313 and the correction fluid. Go over the directions and suggest that the students jot down their observations for question 4 as they notice them.

Extend and Vary the Lesson

- Explore the relationship between the number of spokes in a pinwheel and whether the pinwheel has only rotational symmetry or both point and rotational symmetry. (*If there are an odd number of spokes there is no point symmetry, but if there is an even number of spokes there is both point and rotational symmetry.*) Ask the same question about regular polygons (convex, equiangular and equilateral polygons) of 3, 4, 5, 6 or more sides. What do you see here? *(If there is an odd number* *of vertices, the polygon will look different when upside-down. So there is only rotational symmetry. If there is an even number of vertices, it will have rotational and point symmetry.)*



- Ask the students to make a poster of two-dimensional designs, each illustrating one of the following types of symmetry: only line, only rotational, rotational and point but not line, no symmetry, all three symmetries.
- Tell students that symmetry is often used to elicit an emotional response. For example, the wheel covers of sporty cars frequently have only rotational symmetry to convey a sense of action or excitement. Find examples of symmetry and try to discover the emotion the artist wishes to convey.
- Have students redesign the emblem or logo of their favorite NCAA® basketball team so that it contains one or more types of symmetry.
- To learn how symmetry is used in a variety of fields, an excellent reader might wish to read (or scan) *Symmetry* by Hermann Weyl.

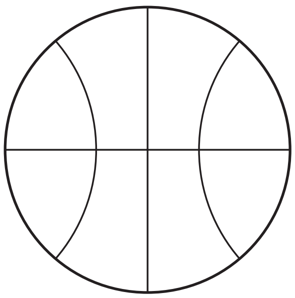
References

Stewart, M. 1988. *Basketball: A History of Hoops.* New York: Franklin Watts.
Weyl, H. 1952. *Symmetry.* Princeton, NJ: Princeton University Press.

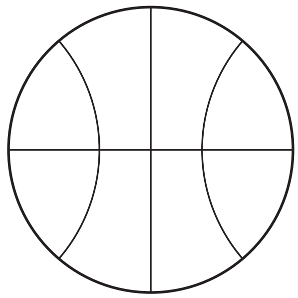
Creating and Destroying Symmetry

Name____ Date___________________

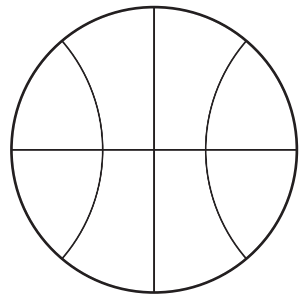
1. The NCAA® basketball is about 30 inches around. This simple diagram of the ball has line, point and rotational symmetry. Change the pictures to produce the desired result.



Only line symmetry

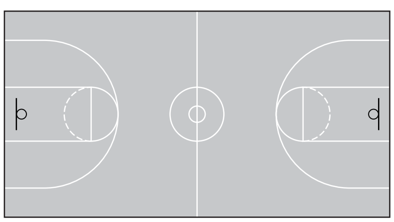


Only rotational symmetry

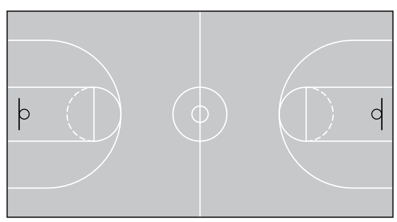


Only point and rotational symmetry

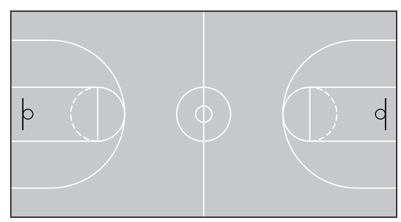
2. Here is a diagram for the floor of an NCAA basketball court. Imagine you have been hired to paint stars on the floor. With the fewest number of stars possible, create the desired symmetry.



Only line symmetry

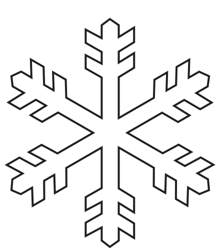


No symmetry

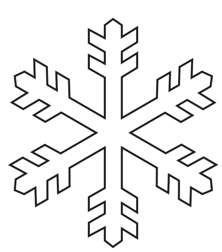


Only point and rotational symmetry

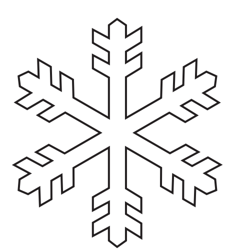
3. The NCAA basketball season extends from October to April. It is considered a winter sport. Change the picture of the snowflake by adding things, subtracting things or both to produce the desired result.



Only line symmetry



Only rotational symmetry



Only point and rotational symmetry

4. Write down some observations you have made about symmetry.

LESSON 3

Free Throws

Students shoot free throws as they drill on ratios and percents.

National Standards: NM.5-8.4, NM.5-8.5, NM.5-8.7
Skills: Finding ratios, converting ratios to decimals, comparing decimals, developing a sense of when to use ratios and when to use decimals
Estimated Lesson Time: 50 minutes

Teacher Preparation

- Duplicate the Free Throws worksheet on page 317 for each student.
- Collect the items listed under Materials.
- Decide where to place the wastepaper basket and the free-throw line. (Experiment at lunch with a few students to see what distance works best for the materials you have on hand.)
- Copy the chart from the worksheet onto the chalkboard.
- (Optional) Use a photocopier to enlarge the box score and then make a transparency or else copy it to the chalkboard.

Materials

- 1 copy of the Free Throws worksheet on page 317 for each student
- 1 pencil for each student
- 1 “basket” (a wastepaper basket, a box, a large-size coffee can)
- About 80 sheets of paper ready for recycling
- Masking tape for marking the edge of the free-throw line and the location of the basket
- 1 calculator for each student
- (Optional) 1 basketball box score from the newspaper

Background Information

Comparing players’ free-throw records is complicated because two players seldom shoot the same number of free throws. If one player shoots 8 free throws and another shoots 10, who is the better player? To decide, we might examine their FTM/FTA ratios, the ratio of free throws made to free throws attempted. Suppose one player has a ratio of 8:9 while the other has 10:12. We could convert both ratios to decimals, which are easier to compare than fractions. (In the newspaper box score these decimals would be recorded in the PCT, or percent, column. The numbers are written as decimals so that three places of accuracy are available. The decimals can be changed to percentages by moving the decimal point two places to the right and adding a percent sign.)

Since 8:9 = $\frac{8}{9}$ a .889 = 88.9% and 10:12 = $\frac{10}{12}$ a .833 = 83.3%, the first player performed slightly better than the second.

Percents do not tell the whole story, though. Two players could have the same percentage, say, 66.7%, yet one player's FTM/FTA could equal 10:15 while the other player's ratio is 2:3. The first player made eight more points for the team.

Introduce the Lesson

Talk about a recent basketball game that might be of interest to the students. Show students the box score from the newspaper and point out that there are three statistics about free throws: FTM (free throws made), FTA (free throws attempted) and PCT (percent). Ask them which statistic tells which player is best. Tell them that today's lesson will show why fans want all three.

Follow These Steps

1. Say that today's lesson simulates shooting free throws in basketball. Go over the rules:
   a. In a real game, players do not make the same number of free throws, so to make the simulation realistic, each person will choose a number between 1 and 10. Using the chosen number, the teacher will apply a "secret rule" (known only to the teacher) and the result will be the number of free throws the player can attempt.
   b. Each player shoots from the "free-throw line" using one sheet of crumpled paper for each attempt.
   c. The game is over after 30 minutes. (This means some students will not be able to shoot.)
2. Distribute the worksheets.
3. Have volunteers come up one at a time. Record their names in the chart on the worksheet and on the chalkboard or transparency. Ask for the number they chose. Use the secret rule (subtract their number from 12) to determine the number of free throws they can attempt. Record this answer under FTA (free throws attempted.)
4. Let each student shoot all his or her free throws one after the other and record the score in the free throws made column, under FTM. Compute PCT as a decimal by dividing FTM by FTA (FTM/FTA = PCT).
5. Continue until the time is up.
6. Discuss the results. Ask who was the best free-throw shooter. Make these points: Two people with the same raw score of baskets made could have different percentages; two people with the same percentage could have different ratios; and many ratios are not reduced in basketball because valuable information, the exact numbers made and attempted, would be lost.

Extend and Vary the Lesson

- Demonstrate how percentages fluctuate. Choose a few college basketball student-athletes. After each game use newspaper box scores and record the number of field goals made and attempted (FGM/FGA). Add these numbers to previous totals to produce cumulative totals for FG, FGA and PCT. Keep a running list of these cumulative totals on a poster on the bulletin board so it is possible to see how PCT changes.
- Learn to read a box score. (See the third grade–fifth grade math lesson 1, "Basketball Box Scores," on page 168.)
- Invite your local college or university's Sports Information Director (SID), the sports statistician from the local newspaper or the team statistician from a local high school or college to talk to the class about collecting sports statistics.

References

This lesson is adapted, by permission, from a lesson by Brian Mulford, University of Illinois student teacher assigned to Urbana Middle School, Urbana, Illinois.

Free Throws

Name____ Date__________________

As each player takes a turn, record the player's name, the number of free throws attempted (FTA), the number of free throws made (FTM) and the percentage both as a decimal (PCT) and as an actual percent.

Name	FTM	FTA	PCT = FTM/FTA (decimal)	PCT as a percent

Answer the following questions:

1. Arrange the names by percent. List the highest value first.

2. Arrange the names by the highest number of free throws made.

3. Who do you think was the best player? Why?____

____

____

LESSON
4

Probability of Competing in College

Students complete drills on the laws of chance as they explore the probability of competing in athletics beyond the high school level.

National Standards: NM.5-8.11, NM.5-8.5, NM.K-4.4, NM.5-8.4
Skills: Finding the probability of an event; writing a probability as a fraction, a decimal or a percent; applying basic probability rules
Estimated Lesson Time: 40 minutes

Teacher Preparation

- Duplicate the Probabilities in Basketball worksheet on pages 321-322 for each student.
- Collect the items listed under Materials.
- Use a photocopier to enlarge the table in the worksheet on page 321 and then make a transparency of the enlargement.
- Make a poster or overhead transparency of the eight basic ideas of probability.

Materials

- 1 copy of the Probabilities in Basketball worksheet on pages 321-322 for each student
- 1 pencil for each student
- 1 calculator for each student
- 1 transparency made from an enlarged copy of the table in the worksheet

Background Information

This lesson provides a quick review of the following basic ideas of probability:

1. An *outcome* is one possible result of an experiment. For example, if the experiment is rolling a die, the outcomes are 1, 2, 3, 4, 5 and 6.
2. An *event* is a set of one or more outcomes. “Rolling a 2” is an event; so is “rolling an even number.” The latter event contains three outcomes: 2, 4 and 6.
3. The *probability of an event,* written *P(event),* equals the number of outcomes favorable to the event divided by the number of possible outcomes. So if the experiment is to roll a die, P(2) = 1/6 and P(even) = 3/6 = 1/2.
4. A probability can be expressed as a fraction, a decimal or a percent.
5. A probability is a number between zero and one, inclusive.
6. A probability of 0 means the event cannot occur.
7. A probability of 1 means the event must occur.

8. P(an event does not occur) = 1 – P(the event occurs).

Introduce the Lesson

Tell students that they will review some elementary rules of probability and use them to discover the probability that a senior on the high school basketball team will make it to a position on an NCAA® basketball team.

Follow These Steps

1. Write *BASKETBALL* on the chalkboard. Tell students to pretend that each of the 10 letters in the word *basketball* is written on a different card, then the cards are shuffled and one card is chosen. Ask, “Which letter has the best chance of being drawn?”
2. Inform the class that each of the 10 cards represents a different outcome. Say that drawing the letter *A* is an example of an event. To find the probability of drawing the letter *A*, we divide the number of ways we can get *A* by the number of possible outcomes that may occur. So P(A) = 2/10 = .20 = 20%. Remind the students that P(A) is read as “the probability of drawing the letter *A*.” Also say that P(A or B) means “the probability of drawing either an A or a B.” Since the number of *A*s and *B*s in the word *BASKETBALL* is a total of four, P(A or B) = 4/10 = 2/5 =.40 = 40 %.
3. Practice finding probabilities with students. Find: P(B), P(S), P(K), P(E), P(T), P(L), P(K or L), P(Z), P(drawing a card with a letter written on it). (*Answers: P(B) = P(L) = 2/10 = .20 = 20%; P(S) = P(K) = P(E) = P(T) = 1/10 = .10 = 10%; P(K or L) = 3/10 = .3 = 30%; P(Z) = 0; P(drawing a card with a letter on it) = 1 = 100%, since only letters can be drawn.*) Tell students that these examples illustrate several rules of probability. Write out and discuss rules 3 through 7. (See the background information on page 318.)
4. Introduce rule 8 with this example. Tell students that P(not drawing an A) is written as P(not A) and that P(not A) = 8/10 = .80 = 80%. Note that P(not A) = 1 – P(A) = 1 – 2/10 = 8/10. Try other similar examples: P(not a vowel) = 1 – P(vowel) = 1 – 3/10 = 7/10. Verify that this answer is correct by counting the letters that are not vowels (7), so P(not a vowel) = 7/10. Tell students that P(not A) = 1 – P(A) is a useful rule for problems like the one in the next example.
5. Put the transparency on the overhead. Solve this problem with the students' help. Find P(a female high school student-athlete becomes an NCAA student-athlete) = _______. (*Answer: 14,400 / 456,900) = .03151674327... ª .0315 ª 3.15%.*) Use this probability to find the P(a female high school student-athlete does *not* become an NCAA student-athlete.) = ____. (*Answer: 1 – .03151674327... = .9684325673... ª .9684 ª 96.84 %.*)
6. Distribute the worksheet on pages 321-322. Circulate and help students as necessary.

Extend and Vary the Lesson

- Go to the Web page **www.ncaa.org**, choose NCAA Educational Outreach and then select Probability of Competing Beyond High School. This page lists the probabilities of competing beyond high school in a variety of other sports including football. Find the probabilities for other sports.

- Discuss why the NCAA makes this data available on its Web page.
- Research the lives of famous athletes to see how they overcame the odds and became successful. Many of the same qualities that produce successful athletes (work ethic, practice, perseverance) also produce good math students.
- For an explanation of how the data in this lesson were collected, go to **www.ncaa.org** and click on site index, then choose probability of competing.

Probabilities in Basketball

Name____ Date___________________

Imagine that the 18 letters in the phrase "NCAA STUDENT-ATHLETE" have each been written on one of 18 cards, the cards have been carefully shuffled and one is drawn. Find the following probabilities as a fraction, a decimal and a percent. Round decimals to the nearest hundredth and round percents to the nearest whole percent.

1. P(N) = ____ = ____ = ____
2. P(C) = ____ = ____ = ____
3. P(A)= ____ = ____ = ____
4. P(T) = ____ = ____ = ____
5. P(vowel) = ____ = ____ = ____
6. P(not a vowel)= ____ = ____ = ____
7. P(N or T) = ____ = ____ = ____
8. P(N, E or T) = ____ = ____ = ____
9. P(Z) = ____ = ____ = ____

Use the information in the table to compute the probabilities in the following problems.

Student-athletes	Men's basketball	Women's basketball
High school student-athletes	549,500	456,900
High school senior student-athletes	157,000	130,500
NCAA® student-athletes	15,700	14,400
NCAA freshman roster positions	4,500	4,100
NCAA senior student-athletes	3,500	3,200

10. Find the probability that a male high school senior wins an NCAA® freshman roster position.

11. Find the probability that a male high school senior becomes an NCAA senior student-athlete.

12. Find the probability that a female high school senior wins an NCAA freshman roster position.

13. Find the probability that a female high school senior becomes an NCAA senior student-athlete.

14. Find the probability that a male high school student-athlete becomes an NCAA senior student-athlete.

15. P(you will have an NCAA roster position next year) = ____________________.

16. P(math teacher will not have an NCAA roster position next year) = ____________________.

Remember that P(an event does not occur) = 1 – P(the event occurs). Use this fact and the information from the preceding table to find the following probabilities.

17. If P(A) = .65 then P(not A) = ____________________.

18. If P(B) = 3/4 then P(not B) = ____________________.

19. P(a female high school senior basketball player does not win a NCAA freshman roster position) = ____________________.

20. P(event occurs) + P(event does not occur) = ____________________.

LESSON
5

Rebounds!

Students shoot for a better histogram as they produce stem-and-leaf plots.

National Standards: NM.5-8.10, NM.5-8.2, NM.5-8.4, NM.5-8.3
Skills: Displaying data with stem plots, reading stem plots
Estimated Lesson Time: 50 minutes

Teacher Preparation

- Duplicate the Rebounds! worksheet on pages 327-328 for each student.
- Collect the items listed under Materials.
- Copy the data listed in step 1 of the lesson to a transparency or to the chalkboard (see page 324).

Materials

- 1 copy of the Rebounds! worksheet on pages 327-328 for student
- 1 pencil for each student
- 1 sheet of graph paper for each student

Background Information

A player shoots. As the ball is released, a struggle ensues on the floor. Players begin *boxing out* (staying between the opponent and the basket), positioning themselves by planting feet, spreading arms to block other players and preparing to spring. Eyes focus on the ball, looking for the opportunity to retrieve it.

When a shot is missed, if one player grabs the ball out of the air and gains possession for his or her team, that player is credited with a *rebound.* If the player is a member of the *offensive* team (the team that made the shot), that player earns an *offensive rebound.* If the player is from the other team, he earns a *defensive rebound.* Should the defensive team cause the ball to go out of bounds in the struggle to gain control after a missed shot, the offensive team gets the ball and is awarded a *team rebound.* Seventy-eight team rebounds were recorded in the 12 regional games of a past NCAA® women's basketball tournament. Once the ball hits the playing floor, rebounds are no longer possible until the next missed shot.

In this lesson, students examine data related to rebounds using *stem plots* (stem-and-leaf plots).

Using stem plots has many advantages. Stem plots rapidly reveal the shape of the data set, allowing us to quickly extract meaning from the data. To determine the best way to display the data, we can swiftly make several different stem plots in succession. Stem plots are easily converted to histograms (just rotate the stem plot 90 degrees counterclockwise) or to other statistical formats. Further, the data values are not lost so the reader can, if necessary, still find the measures of center (the median, the mean and the modes).

Introduce the Lesson

Define the term *rebound* (an attempt by any player to secure possession of the ball after a try for a goal; in a rebound there is no plyer or team control), because even some adults are unsure of the meaning. Determine if the students have an opinion as to whether the winning team generally makes more rebounds than the losing team. Perhaps the winners are so good at shooting baskets that their rebounding total will be lower than the losers'. Tell them that today's lesson uses stem-and-leaf plots (better known as stem plots) to answer that question.

Follow These Steps

1. Display the data on the transparency:

   Rebounds recorded in the 12 regional games of a past NCAA Division I women's basketball tournament:

   41, 33, 48, 37, 27, 37
   33, 43, 34, 34, 34, 37
   28, 41, 35, 26, 35, 29
   32, 48, 28, 49, 34, 46

2. Have students count the data items (24).
3. Locate the smallest and largest data items (26 and 49). Make a stem, starting with the leading digit of the smallest number (2, for 26) and ending with the leading digit of the largest number (4, for 49).

   2 :
   3 :
   4 :

4. Add the leaves, using the numbers appearing in the data. The tens digit is listed before the colon. All units digits are listed after the colon. If the number to be recorded is 41, place a 1 after the 4 : in the stem. The second number is 33, so record a 3 after the 3 : . To record the third number, 48, place an 8 to the right of 4 : 1 as shown.

   2 :
   3 : 3 ¨ Represents 33
   4 : 1 8 ¨ Represents 41 and 48

5. Continue to add leaves until there is one leaf for each data item. Cross off each data item as you record it. The plot should look like this:

   2 : 7 8 6 9 8 ¨ 27, 28, 26, 29, 28
   3 : 3 7 7 3 4 4 4 7 5 5 2 4 ¨ 33, 37, 37, 33, 34, 34, 34, 37, 35, 35, 32, 34
   4 : 1 8 3 1 8 9 6 ¨ 41, 48, 43, 41, 48, 49, 46

6. Rearrange the leaves in numerical order. Keep rows and columns carefully lined up. Recommend that students use graph paper and place one numeral in each square. Count to see if all 24 data items have been recorded.

   2 : 6 7 8 8 9
   3 : 2 3 3 4 4 4 4 5 5 7 7 7
   4 : 1 1 3 6 8 8 9

7. Add a label and the key. Explain that without the key, the reader could not tell if "3 : 2" means 32 or 3.2 or .32 or even a rounded version of 321. (The key can also be written as *Stem value = tens place* or *Leaf value is units place.*)

   Rebounds recorded in the 12 regional games of the 2001 NCAA Division I women's basketball tournament

   2 : 6 7 8 8 9
   3 : 2 3 3 4 4 4 4 5 5 7 7 7
   4 : 1 1 3 6 8 8 9 Key: 3 : 2 = 32

8. The data set (.25 .28 .34 .48 .45 .40 .61) might have the key *3 : 4 = .34* or *stem value = tenths place,* or *stem value = hundredths place.* The stem plot would be arranged like this:

   2 : 5 8
   3 : 4
   4 : 0 5 8
   5 : ¨ Note: This column is blank as no data item began with 5.
   6: 1

9. Outline the advantages of stem plots and demonstrate how rotating the stem plot 90 degrees counterclockwise reveals a bar graph or a histogram. Stem plots are so easy to make that students can try out several versions to see which works best. Explain that the stem could be stretched out in intervals of 5 or 2 (rather than 10). This is very useful for really large data sets where one line might go off the page. Stretched out, the rebound data would look like this:

   Rebounds recorded in the 12 regional games of the 2001 NCAA Division I women's basketball tournament

   2 : 6 7 8 8 9 ¨ Values from 25 through 29 are here
   3 : 2 3 3 4 4 4 4 ¨ Values from 30 through 34 are listed here
   3 : 5 5 7 7 7
   4 : 1 1 3
   4 : 6 8 8 9 Key: 3 : 2 = 32

   Which graph does the best job of displaying the data?

10. Summarize the four important parts of a stem-and-leaf plot: stem, leaves, title and key. Review the steps involved in making a stem plot. Distribute the worksheet on pages 327-328 and go over the first few problems with the students.

Extend and Vary the Lesson

- A useful way to compare two groups of related data is to make a double-sided stem plot. An example is displayed with the answers for this lesson. Go to **www.ncaa.org** and obtain the data for the men's Division I tournament. (Select statistics at the top right of the page; then select men's basketball; then select record books; then select NCAA Men's Basketball's Finest view online.) Compare the number of rebounds made by the winning team and the losing team. Are the results the same for the men as they were for the women?
- Collect data on the members of the class. Ask students to record their gender, the number of people in the vehicle they rode in to school, the number of minutes they spend studying each week, the price they pay for a haircut (it could be zero), the number of relatives that live in the area and so on. Avoid asking for weight or any other variable that might cause embarrassment. Compare the data for the males and females and make stem plots.
- Learn more about displaying data. Read the first few chapters of *The Cartoon Guide to Statistics* by Larry Gonick and Woollcott Smith.

References

Siegel, A.F. and C.J. Morgan. 1996. *Statistics and Data Analysis: An Introduction.* New York: John Wiley & Sons.

Utts, J.M. 1996. *Seeing Through Statistics.* Belmont, CA: Duxbury Press.

Suggested Readings

Gonick, L. and W. Smith. 1993. *The Cartoon Guide to Statistics.* New York: HarperPerennial.

Rebounds!

Name____ Date___________________

Use this stem plot to answer questions 1 through 3.

Rebounding Leaders in Women's Division I Tournament Play as of 2001	
12 : 5 5 5 7	
13 : 0 6	
14 : 2 2 8	
15 :	
16 : 1 2	
17 : 0	Key: 17 : 0 = 170
18 :	
19 : 7	

1. The top five all-time women rebounders are listed below. Look at the stem plot and find the number of rebounds for each player. List the highest value for problem A, then next highest for problem B and so on.

	Games	Rebounds
A. Chamique Holdsclaw, Tennessee	22	____
B. Cheryl Miller, Southern California	16	____
C. Sheila Frost, Tennessee	18	____
D. Val Whiting, Stanford	16	____
E. Venus Lacy, Louisiana Tech	13	____

2. Three rebounding leaders made scores between 140 and 150. What were those scores?

3. How many scores are under 140?

4. Starting with the 1973 season, the men's Division I rebounding champion team has been defined as the one with the highest margins (margin = defensive rebounds – offensive rebounds). Make a stem plot. Remember to include the key and the title. Use graph paper to help line up the numbers. The margin for the men's champion Division I team for each year from 1973 to 2000 is listed on the next page:

Year	Margin	Year	Margin	Year	Margin	Year	Margin
1973	18.5	1980	15.4	1987	11.5	1994	8.6
1974	13.8	1981	12.9	1988	9.9	1995	11.0
1975	12.4	1982	10.4	1999	9.6	1996	11.6
1976	12.2	1983	8.8	1990	10.8	1997	10.9
1977	10.8	1984	9.8	1991	9.3	1998	10.0
1978	16.3	1985	9.1	1992	8.3	1999	10.0
1979	13.8	1986	8.6	1993	11.2	2000	11.7

5. In tournament play, are more rebounds made by the winners or the losers? Display the data as two stretched stem plots. (For each plot, use intervals of 20, 25, 30 and so on.)

Round 2 NCAA® 2001 Women's Basketball Tournament Rebounds
A. Winning Teams: 44, 34, 30, 29, 40 38, 37, 52, 46, 46 38, 37, 53, 35, 37 40
B. Losing Teams: 28, 25, 39, 39, 34 24, 48, 34, 38, 48 24, 35, 27, 27, 43 43

LESSON 6

Sports Equations

Students receive an algebraic assist in solving sports story problems.

National Standards: NM. 5-8.9, NM. 5-8.4, NM. 5-8.1
Skills: Solving linear equations; solving word problems involving ratios, percents or consecutive numbers by writing equations; substituting into a general formula and then solving the equation
Estimated Lesson Time: 40 minutes

Teacher Preparation

- Duplicate the Algebra in Basketball worksheet on page 332 for each student.
- Collect the items listed under Materials.

Materials

- 1 copy of the Algebra in Basketball worksheet on page 332 for each student
- 1 pencil for each student

Background Information

These are the basic steps to solving word problems:

1. Define the variable.
2. Write the question.
3. Solve the equation.
4. Check.
5. Look back. Did you answer the question?

Students usually see the importance of steps 2 and 3, but they often skip steps 1, 4 and 5. Emphasize step 1 by always modeling it. Step 1 makes step 5 easier and making step 5 a habit is good preparation for taking standardized tests. In some multiple-choice tests the value of the variable is often listed as one of the choices along with the correct answer. The correct answer will be the answer to a question such as, “What is the largest of the numbers?” The variable will not represent the largest number. Circling the question makes looking back easier.

You can reinforce the habit of checking by giving partial credit for problems where the student has checked, discovered an error and noted the fact on the test. You must allow ample time for checking. A good rule of thumb is that the teacher should be able to complete and check a 50-minute test in 10 minutes. This means 10 minutes are needed to write the test and the remaining 40 minutes of the class period can be considered thinking time.

Introduce the Lesson

Tell students that today, in small groups, they will review word problems involving ratios, averages, consecutive numbers and a general formula.

Follow These Steps

1. Briefly review the five steps to solving equations.
2. Review problems involving ratios. For example: Find two numbers with a sum of 18 and a ratio of 5 to 4.

Let 5x = the first number and 4x = the second number.

$$5x + 4x = 18$$
$$9x = 18$$
$$x = 2$$
$$(5 \times 2) + (4 \times 2) = 18$$

The question is, "Find two numbers." They are $5 \times 2 = 10$ and $4 \times 2 = 8$.

3. Remind students that algebraically, three consecutive numbers are x, x + 1 and x + 2; three consecutive even numbers are x, x + 2 and x + 4; and three consecutive odd numbers are x, x + 2 and x + 4.
4. Write the basic formula for average on the board.

$$\frac{\text{sum of all the numbers}}{\text{the number of numbers}} = \text{the average}$$

5. Assign students to groups of two or three.
6. Distribute the worksheet, circulate and answer questions.
7. During the last 15 minutes, list the answers on the board so people can get help.

Extend and Vary the Lesson

- Sometimes when the statistics for basketball tournament leaders are listed, only the FG/FGA ratio, the FTM/FTA ratio and the total points are listed. Use this information to write a system of equations to determine how many three-point field goals were made and how many two-point field goals were made. (*Let x = the number of three-point goals made, f = the number of free throws made, y = number of two-point goals and t = the total number of points. The two equations are x + y = total number of field goals (FG) and 3x + 2y + f = t.*)
- In many occupations someone organizes the work so the employee does not have to solve any equations. All that is necessary is to substitute into the equation. Solve 3x + 2y + f = t for x, the number of three-point field goals. ($3x + 2y + f = t$ Æ $3x = t - 2y - f$ Æ $x = \frac{t - 2y - f}{3}$)
- Some coaches have systems for rating players. They plug statistics for the players into an equation and solve. They compare the results and rank the player with the highest score first.

$$y = \frac{(p + 1.5\,r + 2a + 1.5\,s + 2\,b) - (1.5t + 2f + [m - o])}{g}$$

where p = ***p***oints earned
r = number of ***r***ebounds
a = number of ***a***ssists
s = number of ***s***teals
b = number of ***b***locks
t = number of ***t***urnovers
f = number of personal ***f***ouls
m = number of ***m***issed shots
o = number of ***o***ffensive rebounds
g = number of ***g***ames played

Reference

National Collegiate Athletic Association. 2000. *Middle School Madness.* Indianapolis, IN: NCAA.

Algebra in Basketball

Name____ Date____________________

Define the variables, write an equation, solve and check. Remember to look back and make sure you answered the question. Find the total number of points a player makes in a game using this formula: $3x + 2y + f = t$, where x = the number of three-point field goals, y = the number of two-point field goals, f = the number of free throws and t = total points. Answer these questions on a separate piece of paper if needed.

1. Marcus played three games with his basketball team and made a total of 38 points. What was his average number of points per game?

2 . Juanita's favorite basketball player scored 177 points in a tournament for an average of 35.4 points per game. How many tournament games did she play?

3. In the 1986 Division I women's basketball tournament, Drake University's Wanda Ford played two games and made an average of 33.5 points per game. How many points did she make in all?

4. Ricky made eight two-point field goals, one three-point field goal and five free throws in his game at the YMCA. How many points did he make in all?

5. Jackie Stiles, the 2001 NCAA® women's Division I tournament scoring leader, scored 140 points. She made a total of 46 two-point field goals and 42 free throws. How many three-point field goals did she make?

6. Jamal's math teacher watched his junior high basketball game and pointed out the next day that the number of Jamal's free throws and the number of his two-point field goals were two consecutive odd numbers. He had made more two-point field goals than free throws and no three-point field goals. If Jamal made a total of 19 points, how many field goals did he make?

7. Patricia was playing one-on-one with her younger sister. If the ratio of Patricia's baskets to her sister's baskets was 4:3 for a total of 21 baskets, how many baskets did Patricia make?

8. Monique has been keeping track of her points while playing basketball in PE. So far the ratio of her field goals to her free throws is 6:1. She has made no three-point goals. If she has made 39 points, how many field goals has she made?

9. Tenzin plays basketball with the park district. The ratio of his free throws to two-point field goals to three-point field goals was 6:8:1. He has made a total of 50 points. What is his total number of field goals made?

LESSON 7

Go, Math!

Students practice teamwork, cooperation and sporting behavior as they compete in solving review problems in sports math.

National Standards: NM.5-8.4, NM.5-8.5, NM.5-8.6, NM.5-8.7, NM.5-8.8, NM.5-8.9, NM.5-8.10, NM.5-8.11, NM.5-8.12
Skills: Finding the circumference of a circle given the radius, finding averages, finding the probability of an event, finding the percentage given the ratio, drawing lines of symmetry, reading a stem plot, writing equations
Estimated Lesson Time: 45 minutes

Teacher Preparation

- Duplicate the Math Team Competition worksheet on page 335 for each student.
- Duplicate at least one answer sheet for each group (available on page 425).
- Collect the items listed under Materials.
- Assign students to teams. (Use a grade-book program to order the students by their averages, divide the list into the same number of sections as there are teams and put one student from each section on each team. Teams of six, seven or eight students work best. If there are extra students assign some teams an extra, lower-achieving student.)

Materials

- 1 copy of the Math Team Competition worksheet on page 335 for each student
- 1 copy of the answer sheet for each group (available on page 425)
- (Optional) 1 calculator for each student

Background Information

Mathletes are students who complete in math contests. Like athletic events, math contests afford students the opportunity to work hard, achieve their personal best and gain recognition for their efforts. Mathletes and student-athletes benefit from entering contests because they are exposed to more and harder practice sessions, they work in a social atmosphere communicating information and solutions to problems, they learn about teamwork and sporting behavior and they improve in skill level.

The event described in this lesson is based on the eight-person team event in the Illinois Council of Teachers of Mathematics (ICTM) state and regional math contests. This event is designed to promote teamwork and cooperation. Students work together on a set of problems and turn in one answer sheet for the group. They develop a strategy for working together. This set of questions is a review of the lessons in this section and it can be used as a review in math class, as a math club program or in a team practice. For contests, use 20 fairly difficult questions. Make reviews like the test.

Introduce the Lesson

Tell students that they are going to review using the format of a math contest. Announce if there will be a prize (e.g., extra-credit points). Mention that employers are looking for employees who know how to cooperate in groups and work as a team, so this is an opportunity to practice such skills.

Follow These Steps

1. Go over the rules. Explain that students will have 20 minutes to work on the problems and record their answers on the answer sheet. Remind them that they can talk to team members, but they must do so quietly so they do not give away solutions to other teams. Explain that there will be a five-minute warning and a one-minute warning near the end of the contest so that they can get their answer sheets in order. Once time is called, no more answers can be recorded.
2. Outline some possible strategies: Quickly scan and claim problems, work in pairs, work as two groups and compare answers, pick a director who assigns problems or work alone and then compare and record answers after the five-minute warning. Recommend that students confer briefly to decide on a strategy. Remind them to put their names on the answer sheet as soon as it is distributed.
3. Assign the teams. Hand out the answer sheets (found in the Answer Key on page 425.) Distribute the questions facedown.
4. Ask if there are any final questions. Start the contest. Watch the clock. After 15 minutes give the five-minute warning and, four minutes later, give the final one-minute warning.
5. Call time. Collect and grade the papers. Because only answers were recorded, there is no partial credit. Give two points for each right answer. Announce the winning team.
6. Read the answers and answer questions about the problems. Discuss the strategies that students used. Which were more effective?

Extend and Vary the Lesson

- Cooperation and strategy come with repeated practice and experimentation. Design other math reviews based on this format.
- Contact your state math group or the National Council of Mathematics (**www.nctm.org** or 1906 Association Drive, Reston, VA 20191-9988) to learn more about math contests in your area.
- Organize a math team or math club.
- If students keep math diaries, ask them to make an entry about their ideas on cooperation when working in groups.

References

Adapted, by permission, from ICTM State Math Contest Rules for eight-person team.

Math Team Competition

Name____ Date___________________

Unless otherwise noted, reduce fractions, round decimals to the nearest tenth and use 3.14 for π. Use a separate sheet of paper if necessary.

1. How many lines of symmetry are in a perfectly drawn square?
2. Luiz made 14 two-point field goals, five free throws and a three-point field goal. Find his total points for the game.
3. Megan played four basketball games and made a total of 64 points. What was her average number of points per game?
4. Field Goals Made in Tournament Play

   2 : 4 5 9

   3 : 2 2 2 7

   4 : 6 <u>9</u>   Key: stem value is tens place

Use the key to determine the data value represented by the underlined number. (Remember to use the stem.)

5. The circle in the center of a basketball court has a diameter of 12 feet. A square has a side of 12 feet. How much larger is the perimeter of the square than the circumference of the circle?
6. A player makes an assist by passing the ball to a teammate who then makes a basket. Juan made 48 assists in five games. Find his average number of assists per game.
7. Each letter in the word "student-athlete" is written on one of 14 cards that are shuffled and placed facedown. Find the exact probability of drawing a *t*.
8. The ratio of Yaun's free throws made to free throws attempted is 10:15. Find her PCT (percent made) to the nearest tenth of a percent.
9. Which of the following have point symmetry?

   ■

   #

   ✶

   ●

   Z
10. There are one red, two blue and four yellow pencils in Tera's bag. When she reaches in (without looking) to take a pencil, what is the probability that she does not remove a yellow pencil?
11. A team emblem (logo) contains a circle with circumference 9.42 cm. How long is the radius?
12. The ratio of Sandy's three-point field goals to his two-point field goals is 1 to 4. He made five free throws, giving him a total of 27 points for the game. How many two-point goals did he make?